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Modeling Fuzzy Fidelity: Using Microsimulation to Explore Age, Period, and Cohort Effects in Secularization

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Abstract

This article presents a microsimulation that explores age, period, and cohort effects in the decline of religiosity in contemporary societies. The model implements a well-known and previously empirically validated theory of secularization that highlights the role of "fuzzy fidelity," i.e., the percentage of a population whose religiosity is moderate (Voas 2009). Validation of the model involved comparing its simulation results to shifts in religiosity over 9 waves of the European Social Survey. Simulation experiments suggest that a cohort effect, based on weakened transmission of religiosity as a function of the social environment, appears to be the best explanation for secularization in the societies studied, both for the population as a whole and for the proportions of religious, fuzzy, and secular people.

Keywords

demographic projection – religiosity – secularization – microsimulation – cohort effects

1 Introduction

What are the mechanisms that drive secularization in contemporary societies? Under what conditions are populations most likely to experience a decline in religiosity? What role do age, period, and cohort effects play in these processes? These are among the most contested questions discussed by researchers interested in religion and demography. In this article we attempt to contribute to these debates by describing the construction (and reporting on the simulation experiment results) of a microsimulation model designed to simulate processes of secularization hypothesized in a prominent theory of secularization (Voas 2009).

1.1 *Secularization and Fuzzy Fidelity*

The term secularization commonly refers both to the waning power of religious institutions and to the waning of religiosity at the individual level, i.e.: “a decline in the extent to which people engage in religious practices, display beliefs of a religious kind, and conduct other aspects of their lives in a manner informed by such beliefs” (Bruce 2002, p. 3). We focus in this article on secularization at individual level, which involves a drifting away from identifying with a religion, holding supernatural beliefs, attending worship services, praying, and regarding religion as personally important. Here we will use the term “secular” as opposite to “religious” and the term “secularity” to refer to the state of being secular.

Voas (2008) argues that the process of secularization (i.e., the long-term religious decline and the complex of causal connections that promote it) is analogous to the demographic transition (i.e., the shift to longer life expectancy and then low birth rates in the presence of economic development) in a number of respects, not least in that the trends are clear but the mechanisms are not. The “secular transition” comes late in the course of modernization, and it is difficult to slow, stop, or reverse once it begins. Voas subsequently offers a model to illustrate how the seemingly disparate and complex patterns of religious change observed across Europe could all emerge from a common process of secularization. This article aims to replicate two key elements in this theoretical and empirical treatment of “The rise and fall of fuzzy fidelity in Europe” (Voas 2009). The first is a quasi-linear downward trend in average religiosity. Although the levels of religious involvement are very different across Europe (being high in Poland and low in the Czech Republic, for example), decline seems to be proceeding at about the same pace across the continent. The second is the way that the share of the population that is neither fully religious nor wholly secular—a group Voas labels the “fuzzy faithful”—rises and then falls over a period of two centuries or more.

In keeping with the literature on the diffusion of innovations (Kucharavy and De Guio 2011; Rogers 2003), the model assumes that the rise of secularity follows a logistic (S-shaped) trajectory. People do not convert from active religiosity to complete secularity in a single step. The rise in the secular share of the population lags behind the decline in the religious share, which makes it possible for the fuzzy faithful to become a majority. Ultimately, however, the proportion in the fuzzy middle falls as the secular transition continues. Explaining these processes of secularization requires attending to three effects: age, period, and cohort. Age effects change religiosity in individuals at particular points in the life course (as a result, for example, of having children or losing parents). Period effects have an impact on everyone alive at a given time and might be associated with crises such as war, recession, or pandemics. Cohort effects are generation-specific changes that are typically linked to the environment of upbringing and peer interactions in teenage years.

1.2 *The APC Identification Problem*

Because any two of age, current year, and year of birth determine the third, there is no unique way (at least on the surface) to determine which of the three processes explain religious decline in secularizing contexts. This is the so-called APC identification problem, which was first analyzed in terms of the APC accounting model (Mason et al. 1973). For example, cohort effects could be equivalently explained by combining period and age effects. The difficulty in identifying these processes is exacerbated in part because data is available only for a couple of decades. Fortunately, in the specific case of the secularization process, we are not helpless in the face of the APC identification problem. There is now aggregate data spanning over four decades and analyses of this data have led researchers to strongly favor a cohort-replacement explanation of the secularization process; even though it is still logically possible that alternative explanations may have produced the observed patterns (Voas 2009; Voas and Chaves 2016). It is then possible to make a plausibility argument: when cohort effects explain religious decline with decent fit, it is mathematically possible but sociologically implausible that apparently independent age and period effects could be so perfectly synced that they produce the same result (e.g., Voas & Chaves 2016).

But we can rarely be confident that only one or two of these effects are in play with any particular demographic phenomenon, so something else needs to be done to escape the APC identification problem. One approach is to use so-called 'side information' to guide our choice in the set of feasible solutions. However, this approach relies on theoretical assumptions that are rarely justified or verifiable (Reeves 2016). Another is to use non-linear models of these effects as a way around their linear dependence. Assuming that all three are indeed in play, however, one might wonder whether age, period, and cohort effects interact such that hidden constraints might permit optimal explanations. This has led to innovations such as the APC-interaction model (APC-I). Luo & Hodges (Luo and Hodges 2020) use the APC-I to handle a classic instance of interaction effects with the possibility of distinctive interactions between age and period. The APC-I enhancement and correction to the classical APC accounting model is an example of a cautious embrace within sociology and demography of methods capable of handling formally complex systems, which are characterized by non-linear interaction effects, amplifying loops, and dampening processes.

1.3 *Rationale for Using Microsimulation*

The most powerful method for understanding and explaining complex social systems is computational simulation, which can be thought of as an intensification of the move Luo and Hodges made in introducing the APC-I. Computational simulations can give expression to every kind of interaction effect, not just the one type that appears in the APC-I. Moreover, they need not be limited to linear models, unlike the APC-I. They can handle forbidding complexity in terms of time periods, non-linearity of interacting variables, and underlying causal processes. This latter point seems promising to social psychologists, for whom the sociologist's traditional framing of the APC identification problem is an odd abstraction from the concreteness of human minds in which age, period, and cohort effects are merely facets of a complex process of self-evaluation and self-transformation in rich social settings. Methods suited to handling complexity, and computational simulation above all, have enormous potential to tackle seemingly intractable problems, such as the APC identification problem that arises whenever sociologists try to explain population change in secularizing contexts.

The model presented in this paper does not go so far as to articulate a causal architecture of religious change within individual human minds. That is a possibility for computational simulations and one that our research group hopes to realize in due course. The current model has a more modest aim: to implement the potential APC processes as described in the literature and demonstrate the possibility and usefulness of a model that (1) is not based on linear assumptions, as APC models have tended to be; (2) includes all three processes of change operative within the same artificial society; (3) promotes evaluation of the relative importance of those types of change; and (4) simulates up to two and half centuries, from early modernity all the way through the last several decades and onwards into the future yielding population projections for religiosity. Thus, this is a proof-of-concept model, establishing that the vehicle functions well even if its full power remains to be exploited. The conclusion of the analysis is secondary. In fact, assuming the kind of S-curve process of decay in religiosity documented in Voas 2009, the model shows that cohort effects supply the best explanation, which is a conclusion broadly favored within the literature (Idler 2021; McAndrew and Richards 2020; Molteni and Biolcati 2018; Stolz, Biolcati, and Molteni 2021; Voas and Chaves 2016; Brauer 2018). But that result should be understood not as an argument for the greater importance of cohort changes so much as validation of a proof-of-concept model with almost unlimited potential for deeper exploration.

Using agent-based models in this context combines the benefits of top-down (driven by macro-level forces) and bottom-up (driven by individual-level behaviors) analysis. On the one hand, complicated theories about the origins and operation of age, period, and cohort effects can be represented straightforwardly. The influences can wax and wane in non-linear ways, and likewise they can interact with each other. It would be extremely difficult to infer details of such complexity from a statistical model. The simulation can thus be theory-driven and deductive rather than wholly data-driven and inductive. Data have an important role in validating and calibrating a computational simulation, but theoretical considerations are the starting point. On the other hand, the outcome of the simulation ultimately depends on individual-level actions and decisions. The environment can matter a great deal, but the unit of analysis is the agent rather than some impersonal force. Explanation may start and end at the macro level, but it must also operate via the micro level (Coleman 1994). If we want to understand the social or psychological mechanisms at work, we need to track what individual agents do. This focus on agency makes simulation more humanistic than might be immediately apparent (Diallo et al. 2019).

There is significant empirical evidence related to the age, period, and cohort effects that are at work in religious change. For example, panel data from countries where there has not been much aggregate movement away from religion (including highly developed countries such as Israel) can help us to see whether and how religious involvement changes with age or life stage in the absence of secularization (Eisenstein, Clark, and Jelen 2017). No simple story applies universally: we can see clear signs of period effects (with many people drifting away from religion during adulthood) in some countries and not in others, for example. And even where generational replacement appears to have far greater impact than age or time, the size of the generation gaps (i.e., cohort effects) will also rise and fall (Voas 2009; Molteni and Biolcati 2018; McAndrew and Richards 2020; Stolz 2020; Idler 2021; Brauer 2018).

We aim, then, to implement our best conjectures about the proximate mechanisms of religious change in a model to see what trajectories they produce, from the outset of the secularization process to a point centuries later. Models generating outcomes that are at odds with our real-world observations can be rejected. The objective is to identify a small number of models that are consistent with 1) theories about how religiosity is or is not acquired, maintained, and transmitted; and 2) data from societies at different stages in the secular transition. Ideally, we will be able to identify patterns of religious change that apply to many countries, as past work suggests may be possible. We also hope to find models that accommodate exceptions or variations.

Below we provide a description of the APC processes implemented in the simulation.

1.4 *Period Effect Processes*

We conceive two different period-effect processes, one static and one dynamic. In the static process, agents' religiosity decreases every year at a constant rate throughout their life, regardless of starting religiosity, inherited parental religiosity, or age of agent. This static process is capturing latent societal-level factors (e.g. improving education, existential security) that are theorized to encourage decline in religiosity over time (Bruce 2011; Norris and Inglehart 2011; Wildman et al. 2020) and these impact everyone living in the society. In the dynamic process, agents' religiosity decreases throughout the life of the individual, regardless of starting religiosity, inherited parental religiosity, or age of agent, but the degree of change is a function of religiosity. Thus, the absolute decline in an agent's religiosity varies across time, where change is smallest among the most religious and secular individuals, and there is a larger decline for those in the middle of the religiosity spectrum. This dynamic process therefore accounts for highly religious traditions that preserve their religiosity better than others, as well as a reluctance among the nominally religious to reject all religion and become wholly secular (Day 2011; Smith and Denton 2009).

1.5 *Cohort Effect Processes*

Parents transmit their religiosity to their offspring with a bias towards lower religiosity values. We call this a cohort effect, since after the inheritance event, when the individual reaches age 12, their religiosity remains constant. This cohort process is supported by evidence that a consistently large predictor of one's own religiosity is the religiosity of one's parents and there is a net decline in religiosity from parents to children in secularizing societies (Cragun et al. 2018; Min, Silverstein, and Lendon 2012; Storm and Voas 2012; Brauer 2018). We also consider an alternative in which the size of the cohort effect depends on how religious the society is. In this case, agents inherit the religiosity of their parents minus a value that reflects the current secularity of the environment (i.e., the share of individuals classified as seculars). We test which of five different measures provides the best fit, based on the relative frequency in the population of the religious, fuzzy, secular, or non-religious (i.e., secular plus fuzzy), or on the product of the religious and secular shares. The rationale behind this assumptions is that just as in the real world, the social environment in the model changes over time, and the aggregate level of religiosity has an impact on religious transmission and socialization of individuals in adolescence, when their religious identities, beliefs and practices are being formed (Min, Silverstein, and Lendon 2012; Strhan and Shillitoe 2019; Voas and Storm 2021).

1.6 *Age Effect Processes*

It is easily argued that age effects on their own cannot produce religious decline. Although people may become more or less religious as they age, that fact would not alter the average religiosity of a stationary population (Voas 2009). To explain secularization, we require period or cohort effects, or some combination of the two, with age effects having at most a moderating influence. Those might still be significant (if some people return to church while raising a family, for example), but the central question is whether individual-level religious change occurs mostly early in life (especially adolescence and young adulthood) or is spread over much of the life course.

Based on findings from the literature, we devised three different processes by which age affects agents' religiosity, independently of inheritance at age 12. In the first process, agents decrease their religiosity as they become older (Lechler and Sunde 2020). In the second one, the effect is reversed, i.e., the religiosity of agents increases as they become older (Argue, Johnson, and White 1999; Azzi and Ehrenberg 1975; Bengtson et al. 2015; Iannaccone 1998). In the third one, agents decrease their religiosity up to an age at which their religiosity starts to increase again, a U-shape effect (Hayward and Krause 2013). These age effects always occur in combination with period or cohort processes because (as mentioned above) age effects alone can never explain secularization processes.

2 The Fuzzy Fidelity Microsimulation

2.1 *Microsimulation Overview*

The microsimulation explores the way that different APC processes lead to a decrease in religiosity over time in a stationary population. (For specificity, we adapted the initial age structure and vital rates from those for Norway, as described below.) The microsimulation was implemented in AnyLogic 8 University version 8.5.2. We designed this microsimulation mindful of the concerns of social and cognitive scientists of religion, particularly those interested in religious decline. The entities represented in the simulation are human agents characterized by age, generation, and religiosity. The religiosity of agents ranges between 0 and 1. We subdivide this range into three equal intervals, classifying agents as religious (R) if their religiosity (a variable ranging between 0 and 1) is ≥ 0.66, seculars (S) if it is ≤ 0.33, and fuzzies (F) otherwise. During the simulation, the religiosity value of each agent changes according to specific APC processes. These processes are based on theory and evidence about the age, period, and cohort effects that we find in studies of religious change. The overall decrease of agents' religiosity in each of these APC processes is an umbrella estimate representing several factors hypothesized and shown to decrease religiosity in human societies, e.g., religious socialization, existential security, pluralism, education, freedom of expression, etc. (Stolz 2020; Wildman et al. 2020; Gervais, Najle, and Caluori 2021).

Voas (2009) starts from a population that is 95% religious, with only 4% in the fuzzy category. In our view that distribution exaggerates the level of religious commitment in even the most traditional societies; it is more realistic to assume that an appreciable proportion of the population is slightly detached from belief and practice. We therefore assume that, although only 1% of the population qualifies as secular at the beginning of the process (agreeing with Voas), 15% can be regarded as fuzzy. A Weibull distribution with appropriate parameters is well suited to defining our starting point. The overall mean religiosity at the outset is 0.81; within the three categories of religious, fuzzy, and secular, the group means are 0.86, 0.56, and 0.22, respectively. The shares of these groups gradually change from one year to the next, and at the same time the average distribution of religiosity changes in a secular direction.

The initial population is fixed at 1000 agents. The values for age, mortality, and fertility are based on statistics obtained from Statistics Norway (Statistisk Sentralbyrå; https://www.ssb.no/en/befolkning). The starting age distribution follows that of the Norwegian population in 1900. For simplicity, we assume no gender and, to keep the population relatively constant, the total fertility rate is fixed at 1.005 per agent throughout the simulation. Thus, every agent produces an average of 1.005 new agents during the reproductive ages of 15–49, equivalent to fertility of 2.01 children per woman. When turning 12 years old, agents born in the simulation inherit a religiosity value similar to that of their parents (see APC processes). If the parent dies before the agent turns 12, the value inherited is similar to that when the parent was last alive. Further, also for simplicity, we use a constant mortality schedule throughout, with life expectancy of approximately 80 years. In each annual time step, agents experience the following: they age by one year, die or give birth with a probability according to their age, and change their religiosity according to the APC process being applied. In all cases, the change in the agents' religiosity is deterministic and governed by the equations given in each of the following processes.

2.2 *Cohort Effects: Simple and Social Influence*

Cohort processes are supported by evidence showing that a consistently large predictor of one's own religiosity is the religiosity of one's parents and that there is a net decline in religiosity from parents to children in secularizing societies (Cragun et al. 2018; Min, Silverstein, and Lendon 2012; Storm and Voas

2012; Brauer 2018). Following this, in the model, at 12 years old agents inherit the religiosity value of their parents with a bias (eq. 1).

$$REL_{offspring} = REL_{parent} * Bias \qquad \text{eq. (1)}$$

where *REL* is the religiosity value of the offspring and parent, respectively, and *Bias* is a value drawn from the Weibull distribution function of AnyLogic. This function takes two different values: alpha, the shape parameter, and beta, the scale parameter. Its formula is given by equation 2:

$$f(x) = e^{\frac{x^{\alpha}}{\beta}} \qquad \text{eq. (2)}$$

the values of alpha and beta are constrained within specific ranges so the distribution will be skewed and thus the religiosity values of offspring will be on average lower than those of their parents.

We implemented an alternative cohort effect that explicitly incorporates social influence rather than simply a general downward bias. In this case, 12-year-old agents inherit the religiosity of their parents minus a constant (C) multiplied by the proportion in the population of one of the following: a) non-religious, b) religious, c) fuzzies, d) seculars, or e) religious multiplied by seculars. The whole term is then multiplied by *Noise,* a value from a normal distribution with $\mu = 1$ and $\sigma = sd$ (eq. 3). Where *sd* is a parameter determined during the optimization experiments (see below).

$$REL_{offspring} = (REL_{parent} - (C * Prop.\ agent.\ category)) * Noise \qquad \text{eq. (3)}$$

Recall that agents are categorized as religious, fuzzy, or secular depending on whether they are in the upper, middle, or lower third of the religiosity range. The environment changes over time as the population becomes more secular, and transmission of religiosity from parents to children tends to be increasingly affected as aggregate religiosity falls.

2.3 *Period Effects: Static and Dynamic*

We model period effects as the loss of individual religiosity over time. At age 12, agents inherit the religiosity of their parents times some noise (value from a normal distribution with $\mu = 1$, $\sigma = 0.05$). Thereafter, their religiosity declines year by year according to equation 4. This static process captures latent societal-level factors (e.g. improving education, existential security) that are theorized to encourage decline in religiosity over time (Bruce 2011; Norris and Inglehart 2011; Wildman et al. 2020) and impact everyone living in the society.

$$REL_{t+1} = REL_t - Inhibitor \qquad \text{eq. (4)}$$

The value of the inhibitor may be a constant or a dynamic value. When dynamic, the inhibitor is a function of the agent's current religiosity, as shown equation 5. This dynamic process accounts for highly religious traditions that preserve their religiosity better than others, as well as a reluctance among the nominally religious to reject all religion and become wholly secular (Day 2011; Smith and Denton 2009).

$$Inhibitor = A * (REL_t - 0.5)^2 + C \qquad \text{eq. (5)}$$

where REL is the religiosity value of the agent at time t, C is the vertex of the quadratic function (i.e., the maximum value that the inhibitor can take), and A is a constant ($-4*C$) that keeps the boundaries of the quadratic function at 0 (Figure 1). Note that the decrease in religiosity occurs fastest when current religiosity is close to 0.5 and more slowly when the value is near the extremes of 0 or 1. This reflects that the most strongly religious families resist secularizing processes within their children most effectively, and less religious families aren't as successful in religious transmission (cf. Smith 2005). Further, note that in both cases (static and dynamic), when the value of the inhibitor is greater than $Rel_{(t)}$, then $Rel_{(t+1)}$ is set to 0.

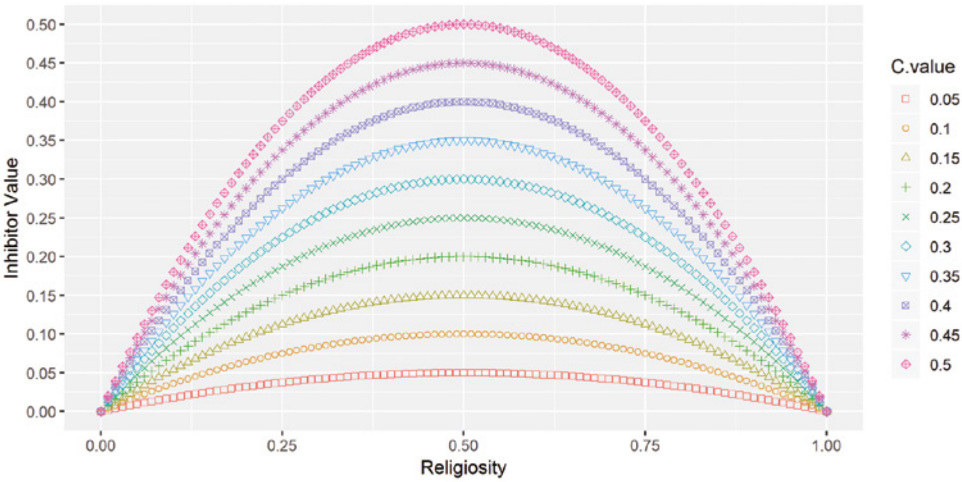

FIGURE 1 Values taken by the inhibitor (y-axis) according to the agent's religiosity value (x-axis) and different values of C (points' color and shape)

2.4 *Age Effects: Religiosity Decreasing and Increasing with Age*

We devised three different processes: (1) agents decrease their religiosity as they become older, (2) agents increase their religiosity as they become older increase, and (3) agents decrease their religiosity up to an age at which their religiosity starts to increase again (U-shape effect). Note that these three effects have an empirical basis (see age processes section). Hence, in the model, when an agent becomes 12 years old and the decrease process is active, the age of the agent modulates the value of the religiosity inhibitor; see equation 6.

$$REL_{t+1} = REL_t - (Inhibitor * Age\ Effect) \qquad \text{eq. (6)}$$

where the inhibitor is a constant value or dynamic value (defined the same way as in eq. 4 and eq. 5, above), and age effect is given by equation 7:

$$Age\ effect = (1 - Age_{standardize})^{\gamma} \qquad \text{eq. (7)}$$

The age of agents is standardized between 1 and 0: 1 when an agent's age is 12 years old and 0 when an agent's age is ≥ 100 years old. Thus, when an agent is 12 years old the age effect is maximum and so is the value of the religiosity inhibitor (Fig 2). Thereafter the age effect decreases as an agent gets older; this decrease is linear or non-linear depending on the value of gamma (γ) (Fig. 2).

Under the influence of the second age effect process, the religiosity of agents increases as they become older. Religiosity starts increasing when an

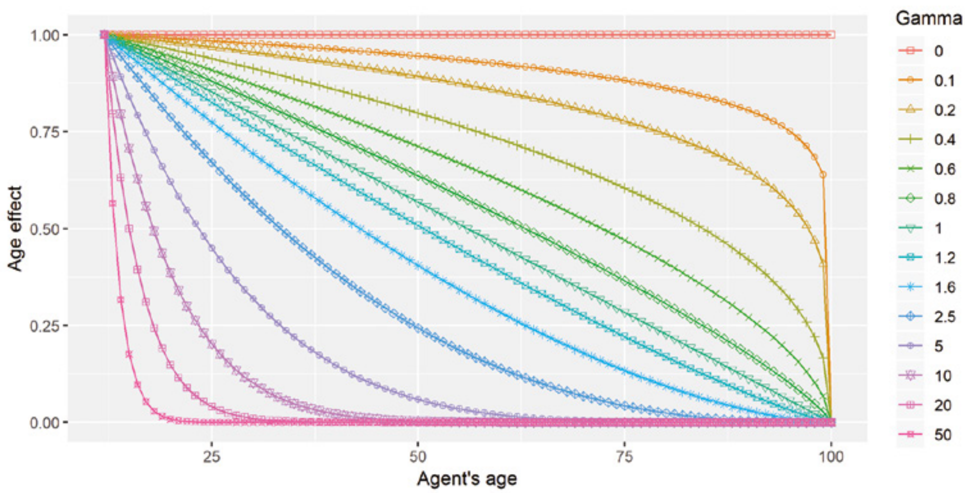

FIGURE 2 Age effect (y-axis) values according to the agent's age (x-axis) and different values of gamma (points' shape and color), for use in the age-effect process where religiosity decreases with age

agent reaches a minimum age, the age of the agent then modulates the value of the religiosity enhancer; see equation 8.

$$REL_{t+1} = REL_t + (Enhancer * Age\ Effect) \qquad \text{eq. (8)}$$

where the enhancer is a constant value, and the age effect is given by equation 9:

$$Age\ effect = (Age_{standardize})^{\gamma} \qquad \text{eq. (9)}$$

In this case, age is standardized between 0 and 1: with 0 being the minimum age at which religiosity starts to increase and 1 being when agents are 100 years old or older. Thus, when an agent reaches the minimum age, the effect of age is minimum and so is the value of the enhancer (Fig 3). Thereafter the age effect increases with age reaching its maximum value at 100 years old. Depending on the value of gamma, the age effect may increase linearly or non-linearly (Fig. 3).

Finally, when the third age effect process is active, religiosity starts decreasing at age 12, according to equation 6 and 7; then, when reaching a minimum age, religiosity starts increasing according to equation 8 and 9.

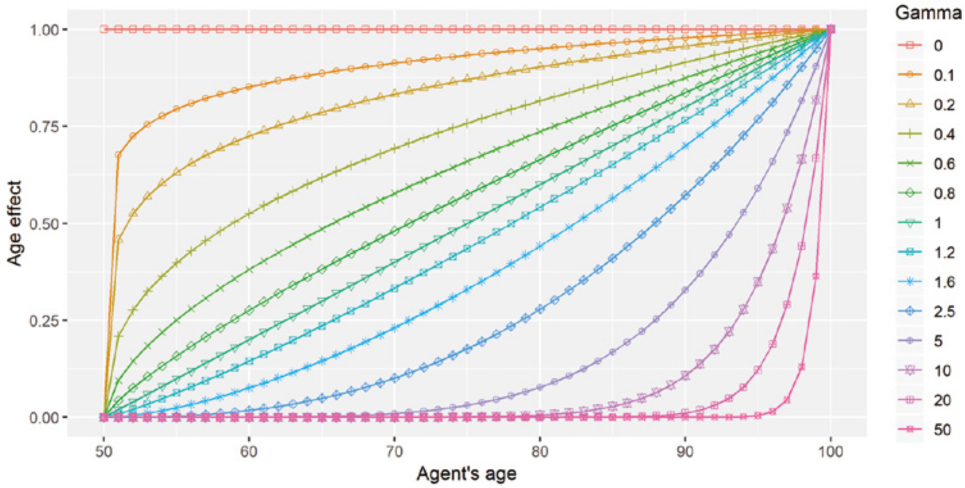

FIGURE 3 Age effect (y-axis) values according to the agent's age (x-axis) and different values of gamma (points' shape and color), for use in the age-effect process where religiosity increases with age. In this example fifty years old is the minimum age at which religiosity starts to increase.

2.5 *Microsimulation and APC Processes Combinations*

Table 1 summarizes the combinations of processes that were implemented in the microsimulation. We defined two types of cohort effects (simple and social-influence, in five variations depending on the nature of the social influence), two types of period effects (static and dynamic), and three types of age effects (decreasing, increasing, and decreasing/increasing with age). From the numerous combinations possible, we selected those that express fundamental options for interpreting demographic process of religious change. Note that the final option in Table 1 includes all five variations of social influence expressing cohort change in religiosity (H, I, J, K, L).

TABLE 1 The eight combinations of age, period, and cohort effects tested

Code	APC process	Equation type and figures	Parameters optimized
A	**Static period effect:** religiosity decay is constant every year	Equation 4: inhibitor is a constant	1. Inhibitor value
B	**Static period effect with age effect (decreasing)**: Religiosity decay depends on inhibitor and decreases with age	Equation 4, 6 and 7; fig 2: Inhibitor is a constant modulated by agent's age	1. Inhibitor value 2. Gamma value (age effect)
C	**Static period effect with U-shape age effect (decreasing)**: Religiosity decreases up to a certain age and then increases—U age effect	Equation 4, 6 and 7; fig 2: Inhibitor is a constant modulated by agent's age. Equation 8 and 9; fig 3: Enhancer is a constant modulated by agent's age.	1. Inhibitor value 2. Gamma value (first age effect) 3. Inflection age, religiosity stops decreasing and starts increasing 4. Enhancer value 5. Gamma value (second age effect)
D	**Dynamic period effect**: decay value is a quadratic function of the agents' religiosity	Equation 4 and 5: Inhibitor is dynamic	1. C value (max inhibitor value)
E	**Dynamic period effect with age effect (decreasing):** decay value is a quadratic function of agents' religiosity and decreases with age	Equations 4, 5, 6 and 7; fig 2: Inhibitor is dynamic and modulated by agent's age.	1. C value (max inhibitor value) 2. Gama value (age effect)

TABLE 1 The eight combinations (*cont.*)

Code	APC process	Equation type and figures	Parameters optimized
F	**Cohort effect (simple):** inheritance is biased towards lower than parental religiosity.	Equation 1: Inheritance with bias.	1. alpha (shape) and beta (scale) values of the Weibull distribution
G	**Cohort effect (simple) with age effect (increasing):** inheritance is biased towards lower than parents' religious values and at a certain age religiosity starts to increase	Equation 1, 2, 8, 9; fig 3: Inheritance with bias. Enhancer is a constant modulated by agent's age.	1. alpha (shape) and beta (scale) values of the Weibull distribution 2. Age at which religiosity starts increasing 3. Enhancer value 4. Gamma value (age effect)
H I J K L	**Cohort effect (social environment):** Religiosity inherited from parents, minus an inhibitor reflecting the religiosity of the population.	Equation 3: Inheritance with noise. Inhibitor is dynamic.	1. C value (max inhibitor value when all the agents are religious) 2. SD, standard deviation of the normal distribution

2.6 *Analysis of Empirical Data*

We needed to evaluate variants of the microsimulation model against data, and we did so using three different approaches. The first and second approach assume that the religiosity decay is logistic and calculate this decay at the cohort and population level respectively. For these calculations, we used the data generated in the model of Voas (see Voas 2009 for details of the data analysis), which assumes logistic decay; this model passed tests against available data so there is a sturdy empirical basis for using it. The third approach assumes that the religiosity decay is linear, we used data from the European Social Survey, extrapolating outwards to cover 200 years. These comparator models are described below.

For the case that religiosity decay is logistic, the projected dynamics of the rise and fall of R-F-S shares over 200 years are shown in Figure 4. The basic concept is that the secular transition starts when the religious share of the population begins to decline, slowly at first, then more rapidly, and slowly again as it approaches a floor. The change in religious share (RS) is given by equation 10:

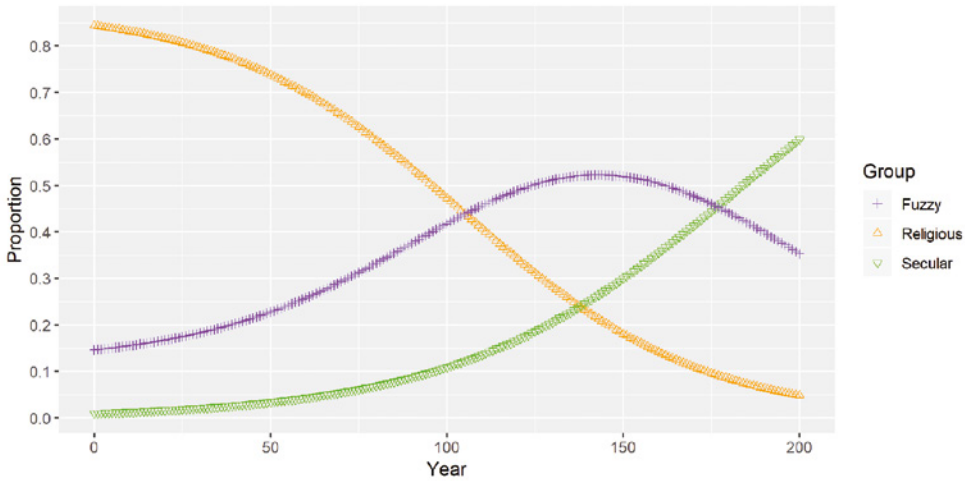

FIGURE 4 Dynamics of the proportions of religious, secular, and fuzzy people at the cohort level. Y-axis represents proportions and x-axis represents time in years

$$RS = \frac{0.88}{1 + e^{-3.15} * e^{0.03 * Year}} \qquad \text{eq. (10)}$$

The wholly secular share (SS) rises from an initial level of just 1%, following the logistic trajectory given by equation 11:

$$SS = \frac{1}{1 + e^{4.6} * e^{-0.025 * Year}} \qquad \text{eq. (11)}$$

The slight lag between these two trends generates the rise of the fuzzy share (FS = 1 – [RS + SS]), which ultimately declines as more complete secularity takes hold (Figure 4). The R-F-S curves relate to birth cohorts, following Voas (2009), and hence we take these graphs as representing 40 5-year cohorts.

The shares of the religious, fuzzy, and secular can be used in conjunction with the average religiosity within each group to calculate the mean religiosity of the whole population. We assume that when the process begins, average religiosity within each category is higher than the midpoint, at 0.86, 0.56 and 0.22 for the religious, fuzzy, and secular groups respectively. During the following two centuries, the shift towards lower religiosity means that these values gradually decline. The largest drop is in the fuzzy group, where average religiosity falls from 0.56 to about 0.46. Multiplying the share of each group by the average religiosity within it gives us the overall mean religiosity by birth cohort (Figure 5).

Additionally, we also calculated the decay of religiosity at the population level. This calculation was done in two different ways. First, using the shares and mean religiosity values of R-F-S agents, we calculated the mean religiosity

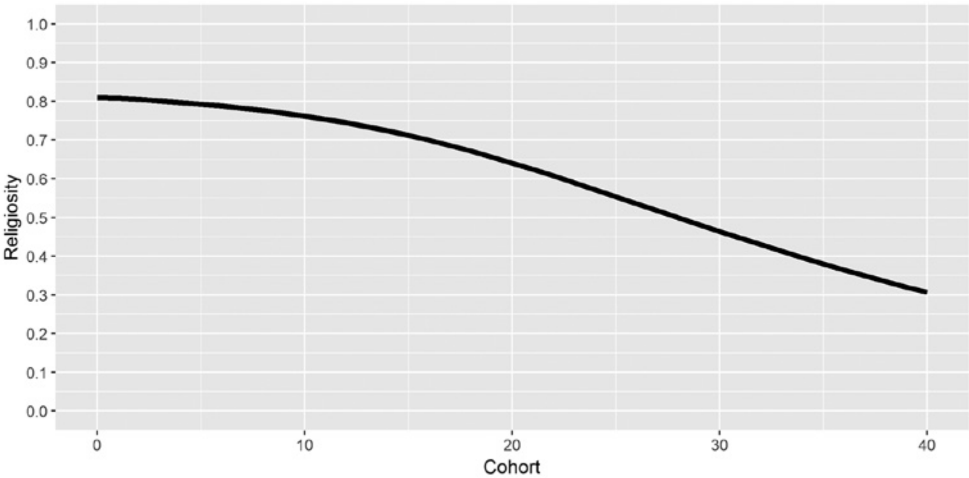

FIGURE 5 Religiosity decay among cohorts

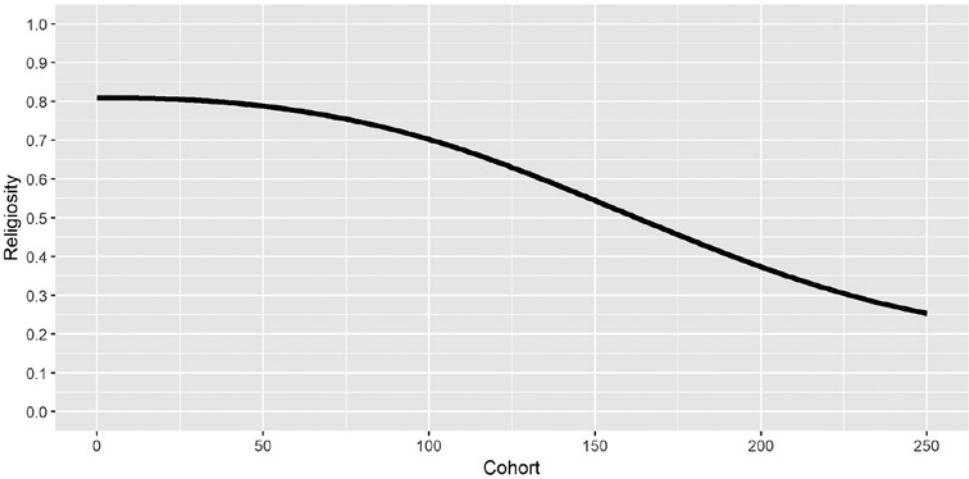

FIGURE 6 Religiosity decay at the population level

of each cohort as described above. We then calculated moving averages, where each average covers ten 5-year cohorts or 50 years of age (to include adults from age 25 to 74). Note that the initial pace of decline is lower because of the inertia from older generations (Figure 6).

For the case that religiosity decay is linear, we used data from the 15 countries that participated in all 9 waves of the European Social Survey (ESS 2018). Detail information on the ESS can be found at (https://www.europeansocialsurvey.org/). First, we calculated a continuous variable called religiosity

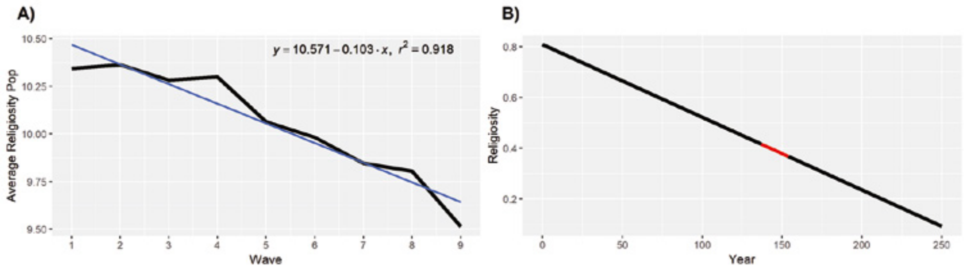

FIGURE 7 A) Religiosity decay at the population level from the 15 countries in the 9 waves of the ESS; B) Religiosity decay at the population level extrapolated from the linear regression in (A) for a period of 250 years; in red, the stretch of religiosity decay calculated from the ESS data in (A)

index using three questions from the ESS. These three questions were also used in the study by Voas 2009: (1) self-declared religiosity (SDR), "Regardless of whether you belong to a particular religion, how religious would you say you are?"; (2) Attendance, "Apart from special occasions such as weddings and funerals, about how often do you attend religious services nowadays?"; and (3) Pray, "Apart from when you are at religious services, how often, if at all, do you pray?". The questions had a 11-, 7- and 7-point scale, respectively; thus, we transformed SDR to a 7-point scale (SDR_7 = 0.6 * SDR_11 + 1). The sum of these answers constituted the religiosity index, ranging from 3 (non-religious) to 21 (very religious). Next, using this religiosity index, we calculated the average religiosity of the population per country and wave, and the average religiosity of the fifteen countries per wave (Table S1 in supplementary information). These calculations show that the average religiosity of the population is decreasing in all countries (Fig 7a). Then, using this data, we performed a linear regression, and found that among these European countries the average religiosity of the population decreases linearly by 0.103 every two years (ESSs were done every two years). Finally, we transformed the religiosity index [3,21] to the religiosity scale used in the model [0,1], and using the initial average religiosity of the population in the model (0.81) as the intercept and the slope from the linear regression, adjusted to the [0,1] scale, we extrapolated the religiosity decay for a period of 250 years. The resulting religiosity decay is shown in Figure 7b. Note that the period covered by the nine ESS waves is only a small portion of the whole range, so the ESS data are consistent with both the logistic-decay and linear-decay hypotheses. Our purpose here is not to evaluate the ESS data but to employ it to generate a credible version of the linear-decay hypothesis that we can use to evaluate the microsimulation alternatives.

2.7 *Optimization of Microsimulation Parameters*

The main goal of the microsimulation is to find, for each combination of APC processes in Table 1, the right parameter values (listed in Table 1) leading to output that mimics the religious decline observed across cohorts or at the population level (Figures 5–7). To do so, we used the optimization engine of AnyLogic v 8.5.2. The optimization engine allows the user to explore many combinations of parameter values with the goal of identifying values that produce the best result, as defined by a particular function. In our case, we try to minimize the residual sum of squares (RSS) between the values obtained from the model and the target religiosity decay curve at: (a) the cohort level, logistic decay with S-shaped curve (Figure 5); (b) the population level, logistic decay with S-shaped curve (Figure 6); and (c) the population level, linear decay (Figure 7b).

To calculate the RSS, we collected the average religiosity (at the cohort or population level as appropriate) from each optimization experiment and compared these values to the corresponding target. For each APC process (Table 1) and target curve, we ran five optimization experiments. We then took the combination of parameter values that produced the lowest RSS and reran the model 100 times, overlaying the target curve with the output of these 100 runs. We thereby established the degree of success with which each APC process could reproduce the target curves for the decline in average religiosity. Similarly, we compared the output of each APC process with the expected changes in R-F-S proportions (Figure 4). Note that the parameters were optimized to produce the best fit with average aggregate religiosity, so the degree to which each proposed solution reproduced the changing breakdown of religious, fuzzy, and secular serves as a form of validation.

3 Results

3.1 *Targeting Logistic Decay of Religiosity at the Cohort Level*

The best fit was produced by the cohort effect taking account of social environment (H–*L* in Table 2). These processes generated RSS values below 0.052, except when the social environment was represented by the proportion of religious population (I in Table 2). Among the different social environments, the best fits were produced when the social environment was represented by the proportion of non-religious (i.e., secular plus fuzzy) or fuzzy agents (H and J in Table 2). Of the other APC processes, the best fits were produced by a static period with U-shaped age effect and a cohort with age effect (C and

TABLE 2 Results of five optimization experiments per APC process targeting the religiosity decay curve at the cohort level

APC processes	RSS values range
A) Static period effect	[0.280–0.298]
B) Static period effect with age effect (decreasing)	[0.133–0.158]
C) Static period effect with U-shape age effect (decreasing)	[0.040–0.133]
D) Dynamic period effect	[0.219–0.241]
E) Dynamic period effect with age effect (decreasing)	[0.082–0.131]
F) Cohort effect (simple)	[0.145–0.156]
G) Cohort effect (simple) with age effect (increasing)	[0.052–0.068]
H) Cohort effect (social environment using proportion of non-religious)	[0.021–0.031]
I) Cohort effect (social environment using proportion of religious)	[0.171–0.206]
J) Cohort effect (social environment using proportion of fuzzies)	[0.022–0.044]
K) Cohort effect (social environment using proportion of seculars)	[0.038–0.052]
L) Cohort effect (social environment using proportion of religious*seculars)	[0.033–0.037]

G respectively in Table 2), but they were not as good as the cohort and social environment effects. All other APC processes produced a much worse fit.

Figure 8 shows the overlay between the cohort target curve and the trajectories of 100 model runs using the combination of parameter values producing the best fit for each of the APC processes. The trajectories in Figure 8 corroborate the results in Table 2: the best fits are produced by the cohort (social environment) effects, particularly when the social environment is represented by the proportion of non-religious or fuzzy agents.

Figure 9 compares the output of these 100 models runs with the dynamics of the R-F-S shares derived from Voas (2009). Here as well, the best fit is produced by the cohort effect when the social environment is represented either by the proportion of fuzzies or non-religious agents. The overlap is not perfect; when using the non-religious proportion as social environment, the fit for the religious category is very good, but less so for fuzzies and seculars. There is a slightly higher proportion of fuzzies around 150 years and a slightly lower proportion of seculars during the first 100 years of the run. In the case of the cohort effect with the fuzzy proportion defining the social

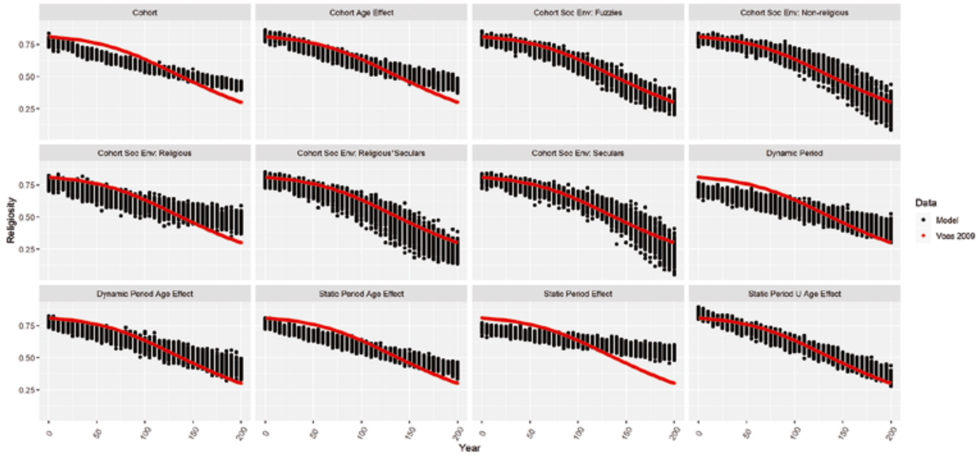

FIGURE 8 Trajectories of 100 model runs for each APC process (black) and the religiosity decay at the cohort level as target curve (red). See text for details

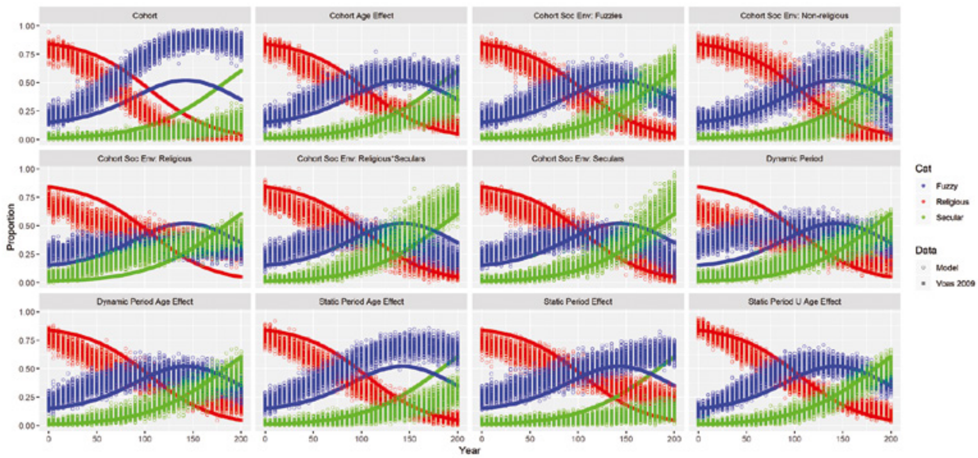

FIGURE 9 Trajectories of 100 model runs for the dynamics R-F-S shares according to each APC process (hollow dots) and the projections according to Voas 2009 (filled squares). Values of the model parameters were optimized by targeting the religious decay at the cohort level. Cat = category.

environment, the proportion of religious individuals appears lower and that of fuzzies higher during the first 100 years of the run. Overall, however, both processes reproduce the R-F-S dynamics well, especially considering that the parameter values of these processes were not optimized to fit these dynamics. Regarding all other APC processes, none of them performs as well as the two just described.

The values of the parameters producing the best fit for each of the APC processes are shown in Table S2 (Supplementary Information). The *C* and *SD* values for the cohort process with social environment represented by the proportion of non-religious agents are 0.172 and 0.058. Hence, when this process is activated, the maximum decrease in religiosity from parent to offspring is a bit higher than 0.172 (depending on the value of *noise*, eq. 3), but only when all agents are categorized as secular or fuzzy. In other words, such a decrease will only happen when nearly the whole population has become non-religious, which takes 200 years. On the other hand, in the cohort process with the social environment represented by the proportion of fuzzies, the values of *C* and *SD* are 0.187 and 0.115 respectively. In contrast to the previous case, the value of *C* and thus the maximum decrease in religiosity from parent to offspring (eq. 3) will never be reached because the proportion of fuzzies is always well short of 1. Here the maximum decay in religiosity is reached after around 150 years, when the proportion of fuzzies is at its peak (Figure 9). Thereafter, the decrease in religiosity from parent to offspring lessens with time.

3.2 *Targeting Logistic Decay of Religiosity at the Population Level*

When targeting the S-shaped decay in religiosity at the population (rather than cohort) level, the best fit was again produced by the cohort and social environment effect (H–L in table 3), particularly when using the proportion of fuzzies or non-religious individuals to characterize the social environments (H and J in Table 3). These processes generated RSS values below 0.052 and as low as 0.012. None of the other APC processes generated a good fit, and in fact all the RSS values were above 0.131 (Table 3). Comparing the 100 model runs with the target curve confirmed the results (Figure S2 in Supplemental Information).

TABLE 3 Results of five optimization experiments per APC process targeting the two religiosity decays at the population level: s-shape and linear decay

APC processes	**RSS values range**	
	S-shape decay	**Linear decay**
A) Static period effect	[0.674–2.232]	[0.005–0.058]
B) Static period effect with age effect (decreasing)	[0.317–0.365]	[0.122–0.348]
C) Static period effect with U-shape age effect (decreasing)	[0.332–0.370]	[0.130–0.145]

TABLE 3 Results of five optimization experiments (*cont.*)

APC processes	**RSS values range**	
	S-shape decay	**Linear decay**
D) Dynamic period effect	[0.414–0.592]	[0.071–0.090]
E) Dynamic period effect with age effect (decreasing)	[0.131–0.184]	[0.195–0.225]
F) Cohort effect (simple)	[0.419–0.505]	[0.329–0.383]
G) Cohort effect (simple) with age effect (increasing)	[0.353–0.422]	[0.345–0.361]
H) Cohort effect (social environment using proportion of non-religious)	[0.029–0.052]	[0.459–0.555]
I) Cohort effect (social environment using proportion of religious)	[0.387–0.716]	[0.530–0.668]
J) Cohort effect (social environment using proportion of fuzzies)	[0.013–0.031]	[0.458–0.552]
K) Cohort effect (social environment using proportion of seculars)	[0.053–0.090]	[0.490–0.613]
L) Cohort effect (social environment using proportion of religious*seculars)	[0.026–0.059]	[0.495–0.583]

Figure S3 (Supplemental Information) shows the overlap between the trajectories of 100 model runs for the R-F-S shares and the projections from Voas (2009). In contrast to the previous results, none of the APC processes produces a good fit (though the same cohort with environment effect solutions are the least unsatisfactory).

3.3 *Targeting Linear Decay of Religiosity at the Population Level*

When targeting linear decay in religiosity at the population level, the best fit was produced by a static period effect (*a* in Table 3). This process generated RSS values as low as 0.005. All other processes performed much worse (Table 3). The results are best illustrated in the overlap between the 100 model trajectories and the linear decay curve (Figure S4 in Supplemental Information). Turning to the dynamics of the R-F-S shares, however, none of the APC processes generated a good fit. All show a large disparity between the model results and the projections by Voas (2009) (Figure S5 in SI).

4 Discussion

This paper presents a computational model as a proof of concept that microsimulations can be used effectively to investigate complex demographic processes such as secularization. Microsimulations can easily express alternative theories of demographic change and enable scholars to evaluate those alternatives against data when available. Microsimulations even offer leverage against the APC identification problem by permitting non-linear interactions among age, period, and cohort effects, after which procedures of the kind demonstrated here allow us to identify the best explanations for a demographic process.

It is important to note that the decline of religiosity in the microsimulation is generated by a simple rule: children receive their religiosity from parents and the transmission of parents' religiosity is moderated by the social environment. This reflects a macro-micro feedback loop, micro in the sense that religiosity is transmitted at the individual level from parents to children and macro because the social environment influences the way both parents and children maintain and pass on their religiosity. Under these conditions, the environment appears to have a homogenous effect in the whole population, i.e., the effect of the environment is the same for all individuals. Interestingly, this process would produce differences between societies if they experience different environmental effects, but would not produce differences within the society, i.e., at the individual level. This is what it is usually found in studies supporting existential security theory, where differences in religiosity are apparent across societies with different GDP, but much less so across individuals of the same society with different socio-economic status (Norris and Inglehart 2011; Stolz 2020).

It is also important to note that the microsimulation is not capable of identifying the triggers of secularization, nor can secularization be stopped in these models. Hence, something else may be needed if we want to explore what may hinder societies from secularizing. However, this issue is out of the scope of our current study; but see (Wildman et al. 2020), where it is considered in a simulation.

Though framed primarily as a proof-of-concept exercise to demonstrate the value of microsimulations in demography of religion and non-religion, the model we have presented is robust enough to make a substantive contribution to the understanding of secularization. When we entertained the hypothesis of linear decay in religiosity, the microsimulation identified a static period effect as the best explanation of the data model, which makes good sense and helps to validate the microsimulation. But a static period effect—and indeed, any of the putative candidates for explaining linear decay of religiosity—could not produce anything close to the correct proportions of religious, fuzzy, and secular people over time observed in the data. This suggests that linear decay is a poor hypothesis and that we are better off with the logistic-decay hypothesis. In light of this, our findings show substantively that Voas' interpretation of cohort replacement, based on weakened transmission of religiosity as a function of the social environment, appears to be an excellent explanation, both for the population as a whole and for the proportions of religious, fuzzy, and secular people.

At the very least, our findings are persuasive support for the claim that secularization is primarily a cohort process. Further exploration of the rich space of model variants possible within this microsimulation could no doubt fine-tune the fit even further and demonstrate how period and age effects play supplementary roles to the dominant cohort effect. That task is for future work.

Supporting Information

Supplemental material is available at https://github.com/ivanpugagonzalez/Modeling-Fuzzy-Fidelity.

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